

Academic Year 2022–2023
PhD in Statistics and Computer Science, Bocconi University

Real Analysis I – Syllabus

Instructor: Hugo Lavenant

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Office: 3-D1-03. I will hold office hours on Thursday afternoon between 16:30 and 18:30 in my office, from 22/09 to 13/10.

Website of the course: All the relevant information and documents are on `blackboard.unibocconi.it`

Schedule: Wednesday 10:15–11:45 and Thursday 14:45–16:15 from September 7 to October 13.

Prerequisites: undergraduate real analysis (set theory, familiarity with $\varepsilon - \delta$ arguments, infinite series, basics of topology). A file with some results that will be assumed to be known can be found on BlackBoard.

Course description. We will cover measure theory and integration. The course will hopefully be useful for students whose research will involve rigorous proofs in probability theory, statistics and functional analysis.

Course outline

1. σ -algebras, measurable functions and measures.
2. Integration.
3. Construction of measures. Lebesgue-Stieltjes measures on \mathbb{R} .
4. Product measures, Fubini's theorems.
5. Signed measures.
6. The Radon-Nykodim theorem.

Objectives. The goal is for the students to understand the concepts of measure theory and to be comfortable with abstract mathematical reasoning. In addition to the understanding of measure theory, the students will be evaluated on their ability to design and write mathematical proofs.

Lecture notes and textbooks. Lecture notes will be distributed at the beginning of the course. They are strongly inspired from the following lecture notes (in French) by J.-F. Le Gall:

- <https://www.imo.universite-paris-saclay.fr/~jflgall/IPPA2.pdf>

You can find material close to what we will cover in the following textbooks (actually in many more):

- R. B. Ash and C. Doléans-Dade, *Probability and Measure Theory*. Academic Press. (Chapter 1 and parts of Chapter 2).
- W. Rudin, *Real and Complex Analysis*. McGraw-Hill Higher Education. (Chapters 1,2,3,6 and 8)

Exercises. Exercises will be posted online. They are of varying difficulty and are quite similar to what will be asked during the final examination.

There will be two exercises sessions where strategies to solve the exercises and solutions will be discussed. They will take place on 29/09 between 08:30 and 10:00, and 20/10 between 14:45 and 16:15.

Examination. There will be a single final *written* examination whose date will be communicated to you soon.