Bocconi University
PhD in Statistics & Computer Science

41000 GRAPH THEORY
Instructors: Luca Trevisan

This course is about algorithms and analytical techniques to study networks, particularly random networks sampled from interesting generative models. The course assumes basic familiarity with directed and undirected graphs, the notions of connectivity and strong connectivity, and of BFS and DFS visits of graphs and their properties.

We will study applications of linear algebraic techniques to graphs, various results about spectral graph theory and spectral algorithms, and we will analyze spectral algorithms for community detection in the stochastic block model and to find planted cliques in random graphs.

Week 1: connectivity, cuts, and spectral graph theory
Lecture 1: Laplacian matrix of an undirected graphs, eigenvalues of Laplacian and connectivity
Lecture 2: Spectral algorithms for graph partitioning and their analysis
Lecture 3: Other connection between Laplacian eigenvalues and combinatorial properties

Week 2: random matrix theory, matrix perturbation theory, and spectral algorithms for random graphs
Lecture 4: spectrum of the adjacency matrix of random graphs, Matrix Chernoff bounds, applications
Lecture 5: spectral algorithms for finding planted cliques in random graphs
Lecture 6: spectral algorithms for community detection in the stochastic block model

Week 3: other matrix norms and semidefinite programming algorithms
Lecture 7: semidefinite programming, Grothendieck inequality, and more on community detection
Lecture 8: semidefinite programming for community detection in the stochastic block model
Lecture 9: semidefinite programming and robustness

Week 4: spectra of graphs, random walks, and other random processes
Lecture 10: the spectrum of Cayley graphs
Lecture 11: expanders, random walks and MCMC algorithms
Lecture 12: percolation